Models of Set Theory II - Winter 2017/2018

Prof. Dr. Peter Koepke, Ana Njegomir Problem sheet 6

Problem 1 (6 points)**.** Let $\langle\langle \mathbb{P}_\alpha, \leq_\alpha, \mathbb{1}_\alpha\rangle \mid \alpha \leq k\rangle$ denote the finite support iteration of the sequence $\langle\langle \dot{\mathbb{Q}}_\alpha, \dot{\leq}_\alpha\rangle \mid \alpha < k\rangle$. Prove the following statements:

(a) If k is finite and $\mathbb{1}_n \Vdash^M_{\mathbb{P}_n}$ "$\dot{\mathbb{Q}}_n$ is σ-closed" for all $n < k$, then $\mathbb{P}_\kappa$ is σ-closed.

(b) The analogue of (a) for k infinite is false.

Problem 2 (6 points)**.** Let $\langle\langle \mathbb{P}_\alpha, \leq_\alpha, \mathbb{1}_\alpha\rangle \mid \alpha \leq \omega\rangle$ denote the finite support iteration of the sequence $\langle\langle \dot{\mathbb{Q}}_n, \dot{\leq}_n\rangle \mid n \in \omega\rangle$. Let $\kappa \geq 2$ be a cardinal in M. Suppose that for each $n \in \omega$,

$$\mathbb{1}_n \Vdash^M_{\mathbb{P}_n} \text{"}\dot{\mathbb{Q}}_n \text{ has an antichain of size } \kappa\text{"}.$$

Show that every $\mathbb{P}_\omega$-generic extension $M[G]$ contains a surjective function $f : \omega \to \kappa$ which is not in M.

Problem 3 (5 points)**.** Let $\langle\langle \mathbb{P}_\alpha, \leq_\alpha, \mathbb{1}_\alpha\rangle \mid \alpha \leq \omega\rangle$ denote the finite support iteration of the sequence $\langle\langle \dot{\mathbb{Q}}_n, \dot{\leq}_n\rangle \mid n \in \omega\rangle$. Prove that if for each $n \in \omega$, $\mathbb{1}_n \Vdash^M_{\mathbb{P}_n}$ "$\dot{\mathbb{Q}}_n$ is atomless" and G is M-generic for $\mathbb{P}_\omega$, then $M[G]$ contains a Cohen real over M.

Problem 4 (3 points)**.** Assume that in M we have that κ is a regular cardinal and $\mathbb{P}$ is κ-cc. forcing notion. Suppose that σ is a $\mathbb{P}$-name such that $\mathbb{1} \Vdash^M_{\mathbb{P}} (\sigma \subset \check{\kappa} \wedge |\sigma| < \check{\kappa})$. Prove that for some $\beta < \kappa$, $\mathbb{1} \Vdash^M_{\mathbb{P}} (\sigma \subset \check{\beta})$.

Please hand in your solutions on Monday, November 20 before the lecture.